Comment on "Experimental demonstration of the violation of local realism without Bell inequalities" by Torgerson et al.*

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Abstract

We exhibit a local-hidden-variable model in agreement with the results of the two-photon coincidence experiment made by Torgerson et al. [Phys. Lett. A 204 (1995) 323]. The existence of any such model shows that the experiment does not exclude local realism.

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In a recent paper [1], Torgerson et al. (hereafter referred to as TBMM) claim that their two-photon coincidence experiment, based on the ideas of Hardy [2], demonstrates the violation of local realism. In particular, they claim that even if three of the four measured probabilities are not exactly zero, as required for Hardy's argument, the results of the experiment still contradict local realism by about 45 standard deviations. We maintain that the experiment refutes only a restricted family of local hidden-variable (LHV) theories containing additional assumptions. To support this point of view, we present a LHV model in agreement with *actual* results of the TBMM experiment (Table 1 in Ref. [1]). The mere existence of any such model (which could be in disagreement with some other measurements with the same experimental set up) proves that local realism is not refuted by the experiment [1]. Then we investigate what supplementary assumptions considered by TBMM are violated in our model.

It has already been proved [3] that no experiment involving only coincidence detection rates may refute the whole family of LHV theories without supplementary assumptions; an argument more specific for the commented experiment follows. Table 1 in Ref. [1] is reproduced if the joint probability that, given one photon in each arm of the TBMM arrangement (see also Ref. [4]), the photon in arm 1 is detected with the polarizer set to the angle θ_1 and the photon in arm 2 is detected with the polarizer set to angle $\bar{\theta}_2 = \theta_2 + \frac{\pi}{2}$, is of the form

$$P_{12}(\theta_1,\, \bar{\theta}_2) = N \left| \cos\theta_1 \cos\theta_2 + \frac{|R|^2}{|T|^2} \sin\theta_1 \sin\theta_2 \right|^2 . \tag{1}$$

Any LHV model must express that joint probability as

$$P_{12}(\theta_1,\, \bar{\theta}_2) = \int P_1(\theta_1,\, \lambda)\, P_2(\bar{\theta}_2,\, \lambda)\, \rho(\lambda)\, \mathrm{d}\lambda\,, \tag{2}$$

where λ denotes collectively the hidden-variables, $\rho(\lambda)$ is the joint probability distribution of these hidden-variables (consequently, $\rho(\lambda)$ must satisfy both $\rho(\lambda) \geq 0$ and $\int \rho(\lambda)\, \mathrm{d}\lambda = 1$), $P_1(\theta_1,\, \lambda)$ is the probability that the photon in arm 1 is detected with the polarizer set to the angle θ_1, and analogously $P_2(\bar{\theta}_2,\, \lambda)$ (therefore $0 \leq P_1(\theta_1,\, \lambda),\ P_2(\bar{\theta}_2,\, \lambda) \leq 1$).

The model we propose has as hidden-variables two unitary vectors, $\boldsymbol{u}_1$ and $\boldsymbol{u}_2$. We define

$$\rho(\lambda)\, \mathrm{d}\lambda = \frac{3}{(4\pi)^2}\, (\boldsymbol{u}_1 \cdot \mathcal{R}\, \boldsymbol{u}_2)^2\, \mathrm{d}^2\boldsymbol{u}_1\, \mathrm{d}^2\boldsymbol{u}_2\,, \tag{3}$$

where $\mathcal{R}$ denotes a rotation of angle φ in the x-y plane and

$$\cos\varphi = |\,R\,|^2 \Big/ |\,T\,|^2 \,, \tag{4}$$

R and T being the reflectivity and transmissivity coefficients of the beam splitter. For the definition (4) we have assumed $|\,R\,| < |\,T\,|$, as in Ref. [1]; if not, the changes $\theta_j \rightarrow \frac{\pi}{2} - \theta_j$ ($j = 1,\, 2$), $|\,R\,| \leftrightarrow |\,T\,|$, would lead to an adequate model. We also define

$$P_1(\theta_1,\, \boldsymbol{u}_1) = C\pi\varepsilon\, f(\boldsymbol{u}_1 - \boldsymbol{r}_1)\,, \tag{5}$$

$$P_2(\bar\theta_2,\, \boldsymbol{u}_2) = C\pi\varepsilon\, f(\boldsymbol{u}_2 - \boldsymbol{r}_2)\,, \tag{6}$$

where $0 < C \leq 1$, $0 < \varepsilon << 1$,

$$f(\boldsymbol{x}) = \begin{cases} (\pi\,\varepsilon)^{-1} & \text{if} \quad |\boldsymbol{x}|^2 \leq \varepsilon \\ 0 & \text{if} \quad |\boldsymbol{x}|^2 > \varepsilon \end{cases}\,, \tag{7}$$

and

$$\boldsymbol{r}_1 = (\sin\theta_1,\, 0,\, \cos\theta_1)\,, \tag{8}$$

$$\boldsymbol{r}_2 = (\sin\theta_2,\, 0,\, \cos\theta_2)\,. \tag{9}$$

With the above definitions and from expression (2), we obtain

$$P_{12} = \frac{3}{16}\, C^2 \varepsilon^2 (\boldsymbol{r}_1 \cdot \mathcal{R}\, \boldsymbol{r}_2)^2 + o(\varepsilon^3)\,. \tag{10}$$

which is in agreement with (1) except for terms of order ε^3. The results of Table 1 in Ref. [1] can be reproduced if ε is smaller than the error in the measurements.

Since the conditions required for Hardy's argument do not strictly occur in real experiments (see (14) in Ref. [1]), additional assumptions and a different argument are required in order to draw a conclusion from the experimental data. TBMM explicitly assume fair sampling. This implies in particular that (a) the photon losses in the polarizers and (b) the efficiencies of photodetectors behind the polarizers are both independent of the polarization of the photons. (b) can be circumvented since in Ref. [1] photodetectors receive photons with the same polarization. However, real polarizers do not satisfy (a). Neither does our model. The equality

$$P_{12}(\theta_1,\, \theta_2) + P_{12}(\theta_1,\, \bar\theta_2) = P_{12}(\theta_1,\, \theta_{20}) + P_{12}(\theta_1,\, \bar\theta_{20})\,, \tag{11}$$

where θ_2 and θ_{20} are two alternative settings for the polarizer 2, is fulfilled by the model only at the lowest order in ε. Violating (11) means that photon absorption of polarizer 2 might depend on the orientation of the polarizer 2 (and similarly for polarizer 1). Since the model violates (11), it also violates Eqs. (2) and (3) in Ref. [1]; but the argument of TBMM (see (15) and the following paragraphs in Ref. [1]) is essentially based on (2) and (3) and therefore cannot be applied.

In addition to our previous comments, we find a more profound criticism to Ref. [1]. The final step of TBMM's reasoning (in particular, their conclusion that the experimental data show a contradiction with local realism of about 45 standard deviations) is based on the following (mathematically incorrect) assumption

$$P_{12}(\theta_{10},\,\theta_{20}) = P_{12}(\theta_1,\,\theta_2)\,P_{12}(\theta_{20}\,|\,\theta_1)\,P_{12}(\theta_{10}\,|\,\theta_2)\,. \tag{12}$$

Under (12) lies a *naïve* idea generally attributed to LHV theories: if a photon is detected behind a polarizer oriented in the θ_1, that is because the photon *had* a polarization θ_1; but what a LHV theory actually says is that the photon has some λ and that $P_1(\theta_1,\,\lambda)$, $P_1(\theta_{10},\,\lambda)$, etc. exist. In LHV, the subset of λ implicated in $P_{12}(\theta_1,\,\theta_2)$ might be different from the subset of λ implicated in $P_{12}(\theta_1,\,-)$ and both might be different from the subset of λ implicated in $P_{12}(-,\,\theta_2)$. Therefore the equality (12) might not be fulfilled and no proof to refute local realism can be based on it.

The indubitable pedagogical value [5, 6] of the argument of Hardy [2], or the possibility of implementing it in many different physical (*gedanken*) contexts [2, 7, 8], does not mean that *actual* experiments based on Hardy's ideas will lead to more conclusive tests to exclude local realism than those based on Bell inequalities [9]. This point is also stressed in Refs. [5, 6]. As Mermin perfectly sums up in Ref. [5], "Hardy's four questions provide a rather weak basis for a laboratory violation of the experimentally relevant inequality" (although "they reign supreme in the *gedanken* realm").

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